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AUTHOR Geary, David C.; And Others
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ABSTRACT

Simple and complex addition problems were presented for true/false verification to 30 undergraduate students to test a general model for cognitive addition. Problems were presented on a microcomputer, with reaction time (RT) and response accuracy recorded. Models for addition were fit to average RT data using multiple regression techniques. These models revealed that subjects were processing complex multicolumn problems columnwise beginning with the units column. Column sums seemed to be obtained through an incrementing process, and subjects exited problems as soon as a column error was encountered. Complex single column multidigit problems were done by "chunking" two digits (summing to 10) and then incrementing the remaining addends beginning with the smallest value digit. These results support the proposed addition model. Implications for future research are discussed. (Author/JN)

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Chronometric Analysis of Complex Addition: Verification of a general model

David C. Geary, Todd D. Little, Keith F. Widaman, Pierre Cormier
Department of Psychology
University of California at Riverside

Presented at the annual meeting of the Western Psychological Association, San Jose, CA, April, 1985

Address Correspondence to: David Geary
Department of Psychology
University of California
Riverside, CA 92521

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Abstract

Simple and complex addition problems were presented for true/false verification to 30 subjects to test the general model for cognitive addition proposed by Widaman, Cormier, & Geary (1985). Models fit to average RT data revealed that subects were processing complex multicolumn problems columnwise beginning with the units column. Column sums seemed to be obtained through an incrementing process, and subjects exited problems as soon as a column error was encountered. Complex single column multidigit problems were done by "chunking" two digits (summing to 10) and then incrementing the remaining addends beginning with the smallest value digit. Results support the model proposed by Widaman et al. (1985). Implications for future research are discussed.

Chronometric Analysis of Complex Addition: Verification of a General Model

Information processing models for cognitive addition in adults include counting based, direct memory access, and memory network models (Parkman & Groen, 1971; Ashcraft, 1982). Groen & Parkman (1972) found that the true sum of simple addition problems, and the smaller addend (MIN) of these problems predicted nearly equal amounts of reaction time (RT) variance. Groen & Parkman (1972) interpreted these results as suggesting that all simple addition facts were retrieved directly from memory, but a counting based (MIN) procedure was used with memory retrieval failure.

Ashcraft & Battaglia (1978), however, found that the true-sum-squared was the best predictor of simple addition RT. Ashcraft & Battaglia, 1978 concluded that there was a network representation for simple addition that resembled a square printed addition table with entry nodes for the digits 0-9 on two adjacent sides. The point of intersection of the two entry nodes in this square matrix represented the correct sum. Entry nodes were assumed to be unevenly spaced; the network was "stretched" in the region of larger sums. Thus, larger sums resulted in considerably longer vector distances, and therefore longer RTs.

The network approach to cognitive arithmetic has successfully predicted performance in adults (Ashcraft & Stayzk, 1981), children (Ashcraft & Fierman, 1982), and for production tasks (Ashcraft, Fierman, & Battaglia, 1984). However, this model has been developed from simple addition and multiplication tasks (Stayzk, Ashcraft, & Hamann, 1982) and has not been generalized beyond complex addition tasks for sums greater than 30 (Ashcraft & Stazyk, 1981).

Widaman, Cormier, & Geary (1985) have recently proposed a general model for

both simple and complex cognitive addtion. The proposed model is multi-staged, and begins with encoding the type of problem, and then encoding the addends (columnwise) beginning with the units place. The units addends are summed either through a network of stored facts or procedurally. If there are more addends in the units column, the process recycles and an additional addend is encoded and summed onto the sum of the first two addends. This process continues until all digits in the units column are summed. The obtained unit sum is compared with the stated sum, and a decision as to correctness is made. If the sum is incorrect the problem is terminated and a "no" response is given. If the units sum is correct, and there are more columns to be added, the process "loops back." The digits for the next column are encoded then summed, etc.. The looping is continued until all columns are completed, or until one column sum is incorrect, at which point the problem is self-terminated.

Widaman et al. (1985) verified this model for simple and complex problems, and found that complex (2-column) problems were done columnwise using a memory network to obtain column sums. Further, a carry operation was highly significant and subjects tended to exit the problem as soon as an error was encountered. However, the most complex problems used by Widaman et al. (1985) contained 2 double-digit addends or 3 single-digit addends. The present study sought to replicate and extend the findings of Widaman et al. (1985) by testing their model with extremely complex problems. More specifically, the present study assessed the usefulness of Widaman et al.'s (1985) general model for multicolumn double-digit addition problems, and for multidigit single-column addition problems.

Method

Thirty undergraduates were presented with 480 addition problems in a one hour session. This global set consisted of two sets of 240 addition problems

each. The first set consisted of four single and multicolumn (1 to 4 columns) sets of double digit problems. The second set consisted of four sets of single column multidigit (2 to 5 digits) problems. Each of the eight problem types consisted of 30 correct and 30 incorrect addition problems.

For incorrect problems, column sums differed from correct sums by ± 1, or ± 2. For complex problems the placement of the error was evenly distributed among the individual column sums, and each problem contained only one error. Similarly, a carry operation (column sum 10) was necessary for about one-half of the cases (per column), and approximately one-half of these cases contained an error. The presentation of problems was random except for the following constraints: 1) there were no more than four consecutive presentations of any one problem type, 2) there were no more than four consecutive presentations of correct or incorrect problems, and 3) same sums or addends were never presented consecutively.

Problems were presented as two sets for true/false verification in column form on an Apple II equiped with a cognitive testing station. One-half of the subjcts were presented with the first (multicolumn) set first. Subjects pressed one response key to indicate if the presented problem was correct and a second key if the presented problem was incorrect. Reaction time and response accuracy were recorded.

Results

Models for addition were fit to average reaction time data using multiple regression techniques. Models fit to RT data included Parkman & Groen's (1971) five counting based models, Ashcraft's (1982) true-sum-squared parameter, and Widaman et al.'s, (1985) reinterpretation of Ashcraft's (1982) model. Specifically, Widaman et al., (1984) argued that Ashcraft's memory network would be better represented by the sum of the digits squared (SSQ), rather than the

square of the summed digits, because each digit squared would represent the seperate vectors on the search through the addition table. For complex problems, models were fit according to the stages outlined by Widaman et al. (1985). Specifically, models were fit in columnwise fashion, with a carry parameter specified if the preceding column sum exceeded 10. A self-terminating parameter was also tested. The self-terminating parameter would represent an executive process (Sternberg, 1982), whereby subjects would "exit" the problem as soon as an incorrect column sum was encountered. Finally, the above (e.g. MIN) simple addition parameters were fit for each column sum. Finally, for multidigit single column problems a chunk (a combination of two digits equalling 10) was fit to RT data.

The error rate was 5.8 percent, and less than one percent of the RT data were deleted as outliers (exceeding 2.58 s.d. of individual RT means). Table 1 and Table 2 present the raw score (RT) regression formulas for the best fitting sets of models for each of the four types of multicolumn two digit problems.

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Insert Table 1 and 2 About Here

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Inspection of Table 1 reveals that Parkman & Groen's (1972) MIN (smallest digit) model with the Truth parameter provided the best fit for simple addition RTs. Ashcraft's (1982) memory network (true-sum-squared) model provided the next best fit for simple addition RTs. All models were fit with a truth parameter representing intercept differences between true and false problems. Consistant with previous findings (e.g. see Groen & Parkman, 1971) false problems were processed more slowly than were true problems.

Inspection of Table 1 and Table 2 reveals that RTs for all complex multi-

column (2 to 4 columns) addition problems were best fit by columnwise parameters (e.g. unitsum) and a carry/self-terminating (e.g. carryst) parameter representing a carry operation if the preceeding column sum was correct and exceeded 10. If the preceeding column sum was incorrect the problem was self-terminated and a "no" excecuted. The carry paramter by itself was highly significant, and the addition of the self-terminating portion increased explained RT variance an average of 2.8 percent.

The best columnwise processing strategy for all complex multicolumn problems was the correct sum of column (e.g. unitsum, tensum etc.), followed by the smallest columnwise addend (e.g. unitmin) of the sum of the digits squared (Widaman, et al., 1985).

Table 3 and Table 4 present the raw score (RT) regression formulas for the best fitting sets of models for each of the four types of single column multidigit problems.

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Insert Table 3 and 4 About Here

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Inspection of Table 3 reveals that, again, the best predictor of simple addition (2 digit one column problems) RTs, was Parkman & Groen's (1971) MIN parameter, followed by Ashcraft's (1982) true-sum-squared parameter. Three single digit addend problems were best predicted by the parameter for the sum of the the largest value addends, and the sum of the smallest addend. The truth parameter was again significant for both problem types suggesting the false problems are processed more slowly than are true problems.

Inspection of Table 4 reveals that the most complex single column multidigit problems were solved using a "chunking" process. The chunk parameter was coded 1

if the sum of any two of the four or five digits was 10, and coded 0 for all other cases. RTs for both four and five digit problems were best predicted by the chunk parameter, and parameters representing the remaining addends individually. The next best model for both problem types represented an addition of addends one at a time straight down the column of digits, beginning with the top digit. False problems were again processed more slowly than were true problems.

Discussion

The results of the present study are consistant with the processing stages for cognitive addition proposed by Widaman et al. (1985). Complex multicolumn addition problems were processed columnwise beginning with the left most column. The carry parameter was highly significant and the hypothesized metacognitive (Sternberg, 1982) self-terminating process was confirmed. Subjects were exiting incorrect problems as soon as a column error was encountered.

Best fitting parameters for individual column sums suggest that subjects were using some type of incrementing process (Groen & Parkman, 1972) to ootain column sums; however, because of the quickness of the incrementation it is unlikely that it was language based (Landauer, 1962). Alternately, subjects may have been moving slowly through a memory network (Ashcraft, 1982) because of the complexity ofa the multicolumn problems (Polltrock & Schwartz, 1984).

The finding that the MIN (Parkman & Groen, 1971) parameter provided the best fit for simple addition RTs in inconsistant with Ashcraft's (1982) memory network for this type of problem. For these subjects an incrementing strategy may have been used for simple addition problems, because these problems were randomly presented with more complex problems which seemed to be completed using an incrementing strategy.

Complex single column multidigit (3 to 5 addends) problems were also processed

according to the stages outlined by Widaman et al. (1985). Specifically, these complex addition problems were processed initially by some combination of two digits from the column, and then the digits were incremented one at a time. Best fitting models suggest procedures used by subjects were systematic. That is, subjects summed the first two digits using a chunking procedure or by adding the two largest value digits, then incrementing the remaining addends beginning with the smallest value digit. Subjects seemed to be using a more sophisticated procedure that a simple summing of addends straight down the column. Although, this type of procedure provided the second best fit for the most complex multidigit problems, suggesting that as the problems became more complex some subjects may have reverted to this less sophisticated strategy. Individual differences research is needed to test this hypothesis.

In all, the processing model proposed by Widaman et al. (1985) was supported by the results of the present study. Future research should focus on individual differences for these complex processing strategies and on the stability of these strategies across times of measurement.

Table 1

Statistical Summaries of Regressions Analyses

One Column Problems

RT = 980.681 + 102.277 (Min) + 231.136 (Truth)
Partial F: 94.54, 23.85
R^2 = .675 $F_{(2,57)}$ = 59.19 MSE = 183.319

RT = 964.821 + 3.056 (Trusum2) + 231.136 (Truth)
Partial F: 77.75, 21.20
R^2 = .634 $F_{(2,57)}$ = 49.47 MSE = 194.410

RT = 777.182 + 53.673 (Trusum) + 231.136 (Truth)
Partial F: 64.72, 19.15
R^2 = .595 $F_{(2,57)}$ = 41.94 MSE = 204.547

RT = 958.112 + 5.300(Unsumsq) + 231.136 (Truth)
Partial F: 45.67, 16.16
R^2 = .520 $F_{(2,57)}$ = 30.91 MSE = 222.715

$\bar{X}$ RT = 1375.807

Two Column Problems

RT = 1526.700 + 71.107 (Unitsum) + 279.800 (Carryst) +
81.392 (Tensum) + 312.419 (Truth)
Partial F: 24.32, 3.47, 52.84, 8.32
R^2 = .742 $F_{(4,55)}$ = 38.82 MSE = 352.945

RT = 2011.368 + 97.11 (Unitmin) + 690.072 (Carryst)
142.234 (Tenmin) + 248.588 (Truth)
Partial F: 15.71, 26.20, 33.56, 4.79
R^2 = .712 $F_{(4,55)}$ = 33.43 MSE = 372.669

RT = 1891.920 + 5.886 (Unsumsq) + 531.878 (Carryst) +
9.237 (Tsumsq) + 248.940 (Truth)
Partial F: 12.67, 12.29, 38.59, 4.74
R^2 = .693 $F_{(4,55)}$ = 30.54 MSE = 384.676

RT = 2050.466 + 1.838 (Unitsum2) + 785.417 (Carryst) +
1.721 (Tensum2) + 42.036 (Truth)
Partial F: 3.57, 19.65, 4.49, 0.09
R^2 = .530 $F_{(4,54)}$ = 15.21 MSE = 476.476

$\bar{X}$ RT = 2645.884

P < .0001 for all models. Note: Min = smaller value digit, Trumsum 2 = Correct sum squared, Trusum = Correct sum, Unsumsq = sum of squared digits, Truth = intercept difference between time and false problems. All others, eqUnitsum, are columnwise parameters.

Table 2

Statistical Summaries of Regression Analyses

Three Column Problems

RT = 1702.741 + 30.347 (Unitsum) + 1,000.614 (Carryst) +
+ 78.846 (Tensum) + 453.927 (Carlost) +
+ 117.749 (Hunsum) + 380.296 (Truth)
Partial F: 1.14, 22.18, 15.71, 5.64, 64.42, 6.57
R^2 = .867 $F_{(6,53)}$ = 56.52 MSE = 472.309

RT = 1969.953 + 2.014 (Unsumsq) + 1242.348 (Carryst) +
9.208 (Tsumsq) + 832.788 (Carlost) +
13.908 (Humsum) + 250.575 (Truth)
Partial F: 0.45, 9.50, 12.27, 17.62, 61.91, 2.85
R^2 = .856 $F_{(6,53)}$ = 51.89 MSE = 490.853

RT = 1549.781 + 2.086 (Unitsum2) + 1431.779 (Carryst) +
2.449 (Tensum2) + 1299.467 (Carlost) +
5.981 (Hunsum2) - 190.632 (Truth)
Partial F: 1.13, 30.85, 2.22, 31.29, 26.53, 1.17
R^2 = .771 $F_{(6.53)}$ = 29.13 MSE = 620.253

$\overline{X}$ RT = 3789.785

Four Column Problems

RT = 1346.252 + 66.314 (Unitsum) + 895.607 (Carryst) +
105.438 (Tensum) + 448.698 (Carlost) +
98.197 (Hunsum) + 544.428 (Carhunst) +
78.751 (Thousum) + 19.738 (Truth)
Partial F: 5.34, 15.20, 20.26, 4.25, 26.61, 6.20, 15.15, 0.01
R^2 = .922 $F_{(8,51)}$ = 74.01 MSE = 508.647

RT = 2129.049 + 32.453 (Unitmin) + 1455.837 (Carryst) +
198.278 (Tenmin) + 823.929 (Carlost) + 221.387 (Hunmin) +
668.838 (Carhunst) + 128.791 (Thoumin) + 401.493 (Truth)
Partial F: 0.60, 63.83, 20.24, 17.59, 29.43, 9.67, 10.33, 5.55
R^2 = .920 $F_{(8,51)}$ = 71.60 MSE = 516.646

RT = 1995.559 + 3.307 (Unsumsq) + 1237.912 (Carryst) +
11.827 (Tsumsq) + 736.978 (Carlost) +
10.817 (Husumsq) + 781.621 (Carhunst) +
8.068 (Thsumsq - 259.277 (Truth)
Partial F: 0.81, 22.86, 9.04, 7.14, 14.01, 8.99, 7.24, 1.46
R^2 = .887 $F_{(8,51)}$ 49.03 MSE = 613.138

$\overline{X}$ RT = 8680.425

P < .0001 for all models. Note: All varibles represent columnwise a logrithms; eg. Tensum = correct sum of tens column.

Table 3

Statistical Summaries of Regression Analyses

Two Single Digit Addend Problems

RT = 869.742 + 133.30 (Min) + 290.614 (Truth)
Partial F: 83.68, 23.42
R^2 = .653 $F_{(2,57)}$ = 53.55 MSE = 232.586

RT = 834.062 + 3.983 (Trusum2) + 290.614 (Truth)
Partial F: 78.87, 22.62
R^2 = .640 $F_{(2,57)}$ = 50.74 MSE = 236.663

RT = 593.271 + 68.705 (Trusum) = 290.614 (Truth)
Partial F: 60.19, 19.51
R^2 = .583 $F_{(2,57)}$ = 39.85 MSE = 258.832

$\bar{X}$ RT = 1356.923

Three Single Digit Addend Problems

RT = 1,049.39 + 38.263 (Maxtwo) + 268.632 (Min) + 237.28 (Truth)
Partial F: 7.69, 121.64, 10.71
R^2 = .796 $F_{(3,56)}$ = 72.80 MSE = 280.746

RT = 1,438.199 + 301.341 (Min) + 237.280 (Truth)
Partial F: 179.01, 9.59
R^2 = .786 $F_{(2,57)}$ = 94.30 MSE = 296.753

RT = 1,122.486 + 163.779 (Mintwo) -2.345 (LargSQ) + 237.20 (Truth)
Partial F: 144.87, 1.29, 8.74
R^2 = .750 $F_{(3,56)}$ = 55.91 MSE = 310.924

$\bar{X}$ RT = 2099.253

P < .001 for all models. Note: Maxtwo = sum of two largest value digits, Mintwo = sum of two smallest digit, Largsq = largest value digit squared.

Table 4

Statistical Summaries of Regression Analysis

Four Single Digit Addend Problems

RT = 1784.080 - 50.384(Chunk) + 232.119(Rem32min) +
123.387(Rem32max) + 312(Truth)
Partial F: 0.05, 25.02, 6.57, 3.16
R = .794 F(4,19) = 18.32 MSE = 402.398

RT = 361.106 + 162.035(Addend1) + 129.597(Addend2) +
122.434(Addend3) + 197.398(Addend4) +
349.138(Truth)
Partial F: 51.12, 33.52, 28.20, 77.84, 7.64
R = .739 F(5,54) = 30.51 MSE = 489.055

Five Single Digit Addend Problems

RT = 1764.412 + 122.256(Chunk) + 405.185(Rem43min) +
125.041(Rem43mid) + 156.750(Rem43max) +
341.130(Truth)
Partial F: 0.17, 40.29, 6.40, 3.62, 3.28
R = .750 F(5,44) = 26.43 MSE = 666.436

RT = 1144.789 + 22.868(Add1sq) + 16.079(Add2sq) +
19.023(Add3sq) + 30.864(Add4sq) +
18.429(Add5sq) + 295.813(Truth)
Partial F: 41.18, 19.73, 26.95, 71.30, 25.60, 2.75
R = .697 F(6,53) = 20.31 MSE = 690.760

$\bar{X}$ RT = 4348.535

$P < .0001$ for all models. Note: Chunk = sum of any two digits equalling 10: Rem32min = smallest remaining addend after a chunk: Rem32max = largest remaining addend after a chunk. Addend1 = first addend etc.: Add1sq = square of the first addend etc..

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